

XXXII. *Comparison between Sir George Shuckburgh and Colonel Roy's Rules for the Measurement of Heights with the Barometer; in a Letter to Col. Roy, F. R. S. from Sir George Shuckburgh, Bart. F. R. S.*

TO COLONEL ROY.

S I R,

Wellbeck Street,
April 20, 1778.

Read May 7,
1778.

SINCE the printing of your ingenious memoir on the subject of measuring heights with the barometer, I have been naturally led to a comparison of your rules and observations with my own^(a); and herein am not more pleased than surprized at the general correspondency of our results, which carries with it the appearance of one and the same series of experiments, rather than of distinct observations made with different instruments, in different countries, and by different persons. That the standard temperature or zero on the scale of the thermometer should be found by each of us to fall in the same point to within one-third of a degree is, I think, truly surprizing; and I doubt not will evince to Mr. DE LUC the strong probability there is of

(a) Vide Phil. Transf. vol. LXVII.

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the necessity of correcting his rules. But although in this essential and fundamental part of the inquiry we agree, there are, nevertheless, some little circumstances wherein we differ; it is the subject of this letter, SIR, to point out to you the degree of our differences, a comparison that I had the pleasure slightly to exhibit to you a few days ago, and which I trust will not be found unimportant to those who may be engaged in these pursuits: if, therefore, you judge these remarks of sufficient moment, I will beg the favour of you to lay them before the Royal Society, as the best means of communicating them to the public.

The two chief causes of our difference are, the expansion of quicksilver and the expansion of air. I shall begin with the equation for quicksilver.

The mean temperature of ordinary barometrical observations, I apprehend, will generally be found to lie between 40° and 70° on FAHRENHEIT's thermometer; now the mean expansion in this range, according to your observation, is ,0323 inch on a column of 30 inches for 10° of heat; by my table it is only ,0304 inch, the difference ,0019 inch is equal to about 20 inches in the result of the height, when the temperature of the two barometers differs 10° , and this may reasonably be expected only in a height of 3000 or 4000 feet. In an obser-

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observation on Mount Ætna, one of the greatest accessible heights in Europe, the difference of temperature at the top and bottom might amount to 30° , and this would occasion a difference of about five feet, which, I apprehend, may be reckoned inconsiderable in a height of 11,000 feet. In fact, in an observation on this mountain by Mr. DES-SAUSSURE it amounted to only $3\frac{1}{2}$ feet. I may add, that your equation makes the computed height less than mine.

I proceed to the expansion of air. Your equation is various according to the circumstances, the difference therefore of our results will, according to the circumstances, be various. The following table will give the quantity of this difference, *viz.* it shews how much your result is + or - mine upon one thousand feet, according to different pressures of the atmosphere and different temperatures. The first column to the left hand contains the mean heat of the column of air between the two barometers; the figures in the horizontal line at top are the mean height of the two barometers, or mean pressure of the atmosphere; the common point of meeting in the different columns gives the difference of our result in feet, according to the respective circumstances.

Mean heat.	Mean height of the two barometers in inches.										
0	30	29	28	27	26	25	24	23	22	21	20
32	0	0	0	0	0	0	0	0	0	0	0
42	+1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
52	+2	0	-2	-4	-6	-8	-10	-12	-14	-16	-17
62	+5	+2	-1	-4	-7	-10	-13	-16	-19	-22	-25
72	+8	+4	0	-4	-10	-12	-17	-21	-25	-29	-33

Thus, if the mean height of the barometer were 27 inches, and the temperature 52° , the difference of the results would be four feet in a thousand; how far, therefore, this is of moment is left to the judgement of the observer. I conclude lastly, SIR, with a comparison of your actual observations made in Great Britain computed after my tables, deduced from a series of observations made in Savoy. I have indeed only collected sixteen of your observations; but as I have chosen such as presented themselves as most proper, either on account of their heights or temperatures, I imagine these will be very satisfactory.

Place of observa- tion.	True height in feet geo- metrically.	By my tables,			By Col. ROY's tables,			Mean temp. of the obs.
		Computed height.	Error in feet.	Error on 1000 feet.	Computed height.	Error in feet.	Error on 1000 feet.	
Station at Weem and top of Bol- frack's Cairn,	1076.5	1074.2	- 2.3	- 2.1	1075.5	- 1.0	- 0.9	58½
Ditto station and top of Dull-craig,	1244.5	1240.6	- 3.9	- 3.1	1244.4	- 0.1	- 0.0	56
Ditto station and South obs. on Schehallien,	2098.0	2096.0	- 2.0	- 0.9	2091.3	- 6.7	- 3.2	52½
Ditto station and West summit of Schehallien,	3281.0 —	3292.5 3261.3	+ 11.5 - 19.7	+ 3.5 - 6.1	3279.0 3252.5	- 2 - 29.5	- 0.6 - 8.9	50½ 46⅞
Carmichael-well and top of Tinto,	1642.5 —	1653.6 1645.0	+ 11.1 + 3.5	+ 6.7 + 2.1	1646.9 1642.7	+ 4.4 + 0.2	+ 2.7 + 0.1	54½ 48
Level of Hawk- hill and small rock on Arthur's Seat,	702.4	704.2	+ 1.8	+ 2.5	703.	+ 1.3	+ 1.9	20½
Base of Hawkhill Observatory and bottom of the small rock,	684.0 —	684.1 685.4	+ 0.1 + 1.4	+ 0.1 + 2.0	686.7 687.0	+ 2.7 + 3.0	+ 3.9 + 4.4	17 68½
Hawkhill garden- door and bottom of the rock on Arthur's Seat,	730.8 —	723.0 745.8	- 7.8 + 15.0	- 10.7 + 20.6	721.8 748.4	- 9.0 + 17.6	- 11.3 + 24.2	29½ 71
Linhouse and East Cairn hill,	1176.6	1182.5	+ 5.9	+ 5.0	1180.0	+ 3.4	+ 2.9	30½
Carnarvon Quay and Moel kilio,	2371.	2397.4	+ 26.4	+ 11.2	2393.3	+ 22.3	+ 9.4	62⅜
Carnarvon Quay and Peak of Snowdon,	3555. —	3563.1 3551.5	+ 8.1 - 3.5	+ 2.3 - 1.0	3551.3 3548.4	- 3.7 - 6.6	- 1.0 - 1.9	56½ 53
		Mean error + 2.0			Mean error + 1.4			

Thus

Thus it seems, that the error of my tables, from a mean of all these observations, amounts to $+\frac{20}{10000}$; of yours, to $+\frac{14}{10000}$: but it must be remarked, that the standard temperature, from whence I compute, is $31^{\circ},24$ of FAHRENHEIT, whereas in your computations it is assumed at $32^{\circ},0$; this difference of $0^{\circ},76$ is equal to $\frac{18}{10000}$ in the correction for the expansion of the air: if then we were to let out from the same zero, *viz.* 32° (which I have proposed, see p. 569. of my memoir, Phil. Transf. vol. LXVII.) the error of my tables, according to your observations, would become only $\frac{2}{10000}$, that of yours remaining $\frac{14}{10000}$. I would by no means from hence conclude, that any preference is to be given to the former, but would say, that in most practical observations, in these countries at least, it is indifferent which method is used. These same comparisons also afford us another piece of information, *viz.* that under similar conditions the density of the atmosphere is the same, whether under the parallel of 46 or 56 degrees of latitude. Till, therefore, more accurate observations than those of Mr. BOUGUER ^(a) can be obtained in the neighbourhood of the

(a) Mr. BOUGUER's observations I hold inadequate in such an inquiry, not only on account of their incorrectness (for they are related only to the $\frac{1}{4}$ of a French line), but particularly inconclusive, allow them all the precision you please, as they were not synchronous; inasmuch, that we are uncertain whether those observations, which should be corresponding in point of time, were made within six months of each other: and with regard to the temperature of some of them

the equator, I should be extremely cautious how I admitted a latitudinal equation; nor do I think the single observation, related in *The Voyage towards the North Pole*, of sufficient authority itself to establish such a theory upon ^(b).

them we are still more at a loss, having a range of no less than 38° to assume it in (vide *Voyage au Perou*, p. 29.). The mean, however, of his five observations, according to my computation, would bring the zero of the scale to about 40° of FAHRENHEIT: but till it can be proved, that in this uncertainty of 38° we have, in fixing upon the mean, got the true temperature to within 8° ; and also, that during many months the barometer in the same place had never altered a single line: till then, I say, no fair conclusion, in point of theory, can be drawn from these experiments, for with such supposed errors or variations the Peruvian observations will agree exactly with those in this climate.

(b) It seems extraordinary, that the heights of the quicksilver, observed on the sea-shore with one of Mr. RAMSDEN's barometers, should differ a quarter of an inch from the height of the same observed the same day, and almost at the same hour, with Mr. NAIRNE's marine barometer on board a ship (vide p. 135 and 148. of *The Voyage towards the North Pole*). This difference, therefore, I think remains to be explained, if the experiment is to be made use of in this inquiry; for this same marine barometer, compared a few days ago at Mr. IBBETSON's, secretary to the board of longitude, with one of my own, that I used in Savoy, agreed with it to within 0,04 inch. It may also be remarked, that Lord MULGRAVE's observation in lat. $79^{\circ} 44'$ brings the zero to about 64° ; so that between the lat. 56° and 79° (equal 23°) the zero of the scale moves through a space of no less than 32° ; whereas, between the lat. 40° and 56° it is perfectly stationary, at least as to sense, having altered only one-third of a degree: which great want of proportion, I think, is of itself some argument against the existence of such a latitudinal equation. Moreover, Mr. BOUGUER's observations brings the zero of the scale on the same side of 32° with Lord MULGRAVE's, viz. to about 40° , so that, if any deduction is to be drawn from these observations, it is, that the air, both at the equator and at the pole is heavier than in middle latitudes. which is no very probable conjecture, and, I apprehend, more than is intended to be proved. I should have asked pardon for the freedom with which I have discussed this inquiry, were I not assured that the acquaintance and friendship of my Lord MULGRAVE, which I have experienced for some years, will lead him to attribute it to its proper motive.

I shall

I shall now beg leave to conclude with what I flatter myself will not here appear improper, a new rule for reducing the observations, and which I hope will be found particularly commodious, as it requires no logarithms, nor any other than the following short table, which may be engraven upon the scale of a thermometer, and therefore, always accompanying the instrument, will serve for computing the observations upon the spot, (if the height should not exceed 4000 or 5000 feet) which, I apprehend, will frequently be found very satisfactory.

Ther.	Feet.	The value of one-tenth of an inch of quicksilver on the barometer, expressed in feet in the atmosphere when the barometer stands at 30 inches, according to the different temperatures.
32	85.86	
35	87.49	
40	88.54	
45	89.60	
50	90.66	
55	91.72	
60	92.77	
65	93.82	
70	94.88	
75	95.93	
80	96.99	

The adjoined table gives the value of $\frac{1}{10}$ th of an inch on the barometer in feet in the atmosphere, when the quicksilver stands at 30 inches, for every five degrees of temperature from 32° to 80° ; and for any other height of the barometer it will be in the inverse ratio of that height to 30 inches. Thus, let A be the mean height of the two barometers in inches; α the difference of the two barometers in $\frac{1}{10}$ ths of an inch; β the number taken out of the adjoined table; x the height in feet; we have then the following expression, $\frac{30 \alpha \beta}{A} = x$, the height required.

I have the honour to be, &c.

